

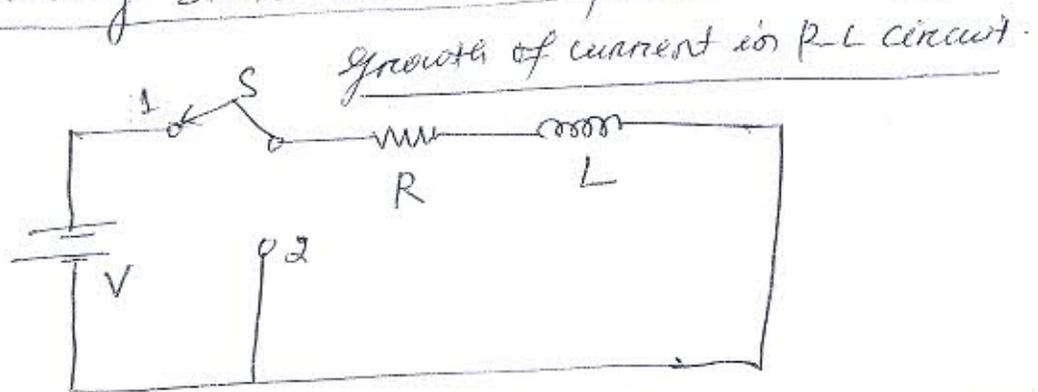
## Solution of First and Second order Networks :->

\* Solution of first and second order differential equations for series and parallel R-L, R-C, R-L-C circuits :->

1st order circuit. It contains resistance and one energy storing element i.e. one inductor or capacitor.

-> This first order circuit, during its transient state of operation, is governed by first order linear differential equation.

D.C. steady state solutions of R-L circuit.



Consider some instant  $t$  seconds after the voltage is applied.

$i$  = current flowing through the circuit at instant  $t$  seconds

$\frac{di}{dt}$  = rate of growth of current at this instant

$V_R = iR$  = voltage across R

$V_L = L \frac{di}{dt}$  = voltage across L.

Applying KVL

$$V = Ri + L \frac{di}{dt}$$

$$\Rightarrow \frac{di}{dt} + \frac{R}{L}i = \frac{V}{L} \quad \text{--- (1)}$$

Equation (1) is a non homogeneous differential equation.  
 Solu<sup>n</sup> of this equ<sup>n</sup> is

$$i = e^{-\frac{R}{L}t} \int e^{\frac{R}{L}t} \cdot \frac{V}{L} dt + Ke^{-\frac{R}{L}t} \quad \text{--- (2)}$$

$$\Rightarrow i = i_p + i_c.$$

Solution of equ<sup>n</sup> (2) is

$$\boxed{i = \frac{V}{R} + Ke^{-\frac{R}{L}t}} \quad \text{--- (3)}$$

At  $t = 0^+$  (just after switching), equ<sup>n</sup> (3) becomes

$$0 = \frac{V}{R} + Ke^{-\frac{R}{L} \cdot 0} = \frac{V}{R} + K$$

$$\Rightarrow K = -\frac{V}{R}$$

Putting in equ<sup>n</sup> (3) we get

$$i = \frac{V}{R} - \frac{V}{R} e^{-\frac{R}{L}t}$$

$$\Rightarrow i = \frac{V}{R} (1 - e^{-\frac{R}{L}t})$$

$$\Rightarrow i = I_0 (1 - e^{-\frac{t}{\tau}})$$

$$\text{--- (4)} \left\{ \begin{array}{l} \text{where } I_0 = \frac{V}{R} = \text{maximum current or steady state current} \\ \tau = \frac{L}{R} = \text{time constant} \end{array} \right.$$

Equ<sup>n</sup> (4) is called Helmholtz equation for growth of current in R-L circuit.

If we put  $t = z = \frac{L}{R}$  in equ<sup>n</sup> (4) we get

$$i = I_0 (1 - e^{-1}) = 0.632 I_0 = 63.2\% \text{ of } I_0$$

If we put  $t = \infty$  in equ<sup>n</sup> (4) we get

$$i = I_0 (1 - e^{-\infty/z}) = I_0$$

Thus current in R-L circuit would attain maximum value ( $I_0$ ) only after infinite time.

Voltage drop across inductor is  $V_L = L \frac{di}{dt}$

$$\Rightarrow V_L = L \cdot \frac{d}{dt} [I_0 (1 - e^{-t/z})] = L \cdot \frac{d}{dt} [I_0 - I_0 e^{-t/z}]$$

$$\Rightarrow V_L = L \left[ \frac{d}{dt} I_0 - \frac{d}{dt} I_0 e^{-t/z} \right] = L \left[ 0 + \frac{1}{z} I_0 e^{-t/z} \right]$$

$$\Rightarrow V_L = L \left[ \frac{1}{L/R} I_0 e^{-t/z} \right] = I_0 R e^{-t/z}$$

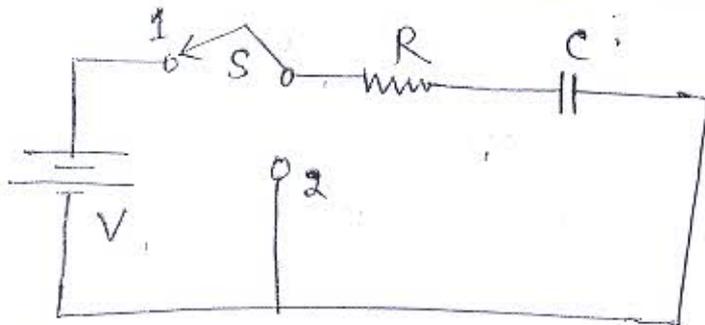
$$\Rightarrow \boxed{V_L = V e^{-t/z}} \quad \text{where } V = I_0 R$$

Voltage drop across resistor is

$$V_R = iR = I_0 (1 - e^{-t/z}) R = V (1 - e^{-t/z})$$

→ In transient period voltage across resistor exponentially rising and voltage across inductance exponentially decaying. Once the transient dies out within a short time then steady current ( $I_0 = \frac{V}{R}$ ) remains in the circuit.

D.C. Steady state Solution of  
R-C circuit.  $\Rightarrow$  Case-I. Charging of R-C circuit.



Consider a R-C circuit connected in series with a battery of voltage  $V$  and a switch  $S$ .

$\rightarrow$  Initially capacitor is uncharged and voltage across it is zero. ( $V_C = 0$ )

$\Rightarrow V_R = V \rightarrow$  Whole supply voltage.

Initial current in the circuit  $I_0 = \frac{V}{R}$

$\rightarrow$  As current flows in the capacitor, starts charging and capacitor voltage  $V_C$  increases.

$\rightarrow$  Consider some instant  $t$  second after the voltage is applied.

$i \rightarrow$  Current flowing through the circuit at instant  $t$  seconds.

$V_R = Ri =$  voltage <sup>drop</sup> across  $R$

$V_C = \frac{1}{C} \int i \cdot dt =$  voltage across  $C$ .

Applying KVL,  $Ri + \frac{1}{C} \int i \cdot dt = V$

Differentiating both sides w.r.t time  $t$ , we get

$$R \frac{di}{dt} + \frac{i}{C} = 0 \Rightarrow \left[ \frac{di}{dt} + \frac{1}{RC} i = 0 \right] \quad (1)$$

(3)

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Equation (1) is a homogeneous equation.  
 Solution of this eqn (1)

$$i = k e^{-\frac{t}{RC}} \quad \text{--- (2)}$$

where  $k$  is a constant, whose value can be calculated from the initial condition.

$$t = 0^+$$

$$\text{Current } i(0^+) = \frac{V}{R}$$

Eqn (2) becomes.

$$\frac{V}{R} = k e^{-\frac{0}{RC}}$$

$$\Rightarrow \boxed{k = \frac{V}{R}}$$

putting this value of  $k$  in eqn (2)

$$i = \frac{V}{R} e^{-\frac{t}{RC}}$$

$$\Rightarrow \boxed{i = I_0 \cdot e^{-\frac{t}{\tau}}} \quad \text{--- (3)}$$

where  $I_0 = \frac{V}{R} =$  maximum current in the circuit.

$\tau = RC =$  Time constant or Capacitive time constant.

Voltage across the resistor.

$$V_R = iR = I_0 R e^{-\frac{t}{\tau}} = V e^{-\frac{t}{\tau}} \quad \text{--- (4)}$$

Voltage across the capacitor  $\Rightarrow$

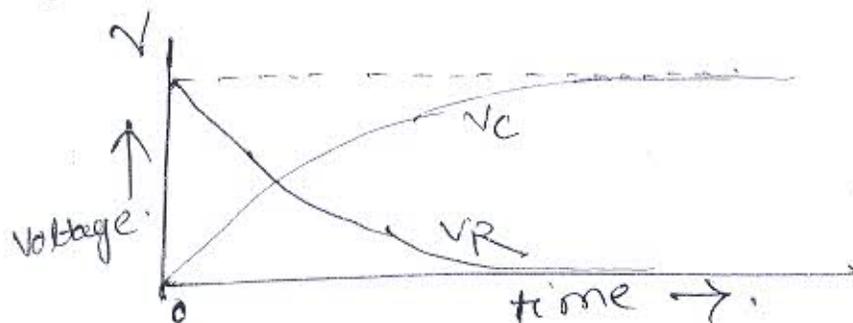
$$V_c = \frac{1}{C} \int i \cdot dt = \frac{1}{C} \int_{t_0}^t e^{-t/\tau}$$

$$\Rightarrow \boxed{V_c = V(1 - e^{-t/\tau})} \quad \text{--- (5)}$$

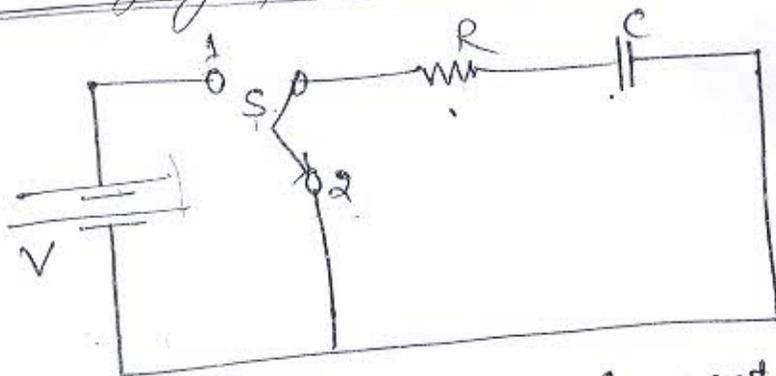
$\rightarrow$  The charge stored in capacitor during charging is  $q = CV_c = CV(1 - e^{-t/\tau}) = Q(1 - e^{-t/\tau})$

$\rightarrow$  If we put  $t = \tau = RC$ , in equ<sup>n</sup> (5) we get.

$$V_c = V(1 - e^{-1}) = 0.632V = 63.2\% \text{ of } V$$



Discharging of RC circuit :->



Let  $i$  = discharging current at any instant.

Applying KVL

$$Ri + \frac{1}{C} \int i \cdot dt = 0$$

Differentiating both sides w.r.t time ( $t$ ), we get

$$R \frac{di}{dt} + \frac{i}{C} = 0 \quad \text{--- (6)}$$

Equ<sup>n</sup> (6) is a homogenous differential equ<sup>n</sup>.  
Sol<sup>n</sup> of this equ<sup>n</sup>.

$$i = Ke^{-\frac{t}{RC}} \quad \text{--- (7)}$$

where 'K' is a const., whose value can be calculated from initial condition.

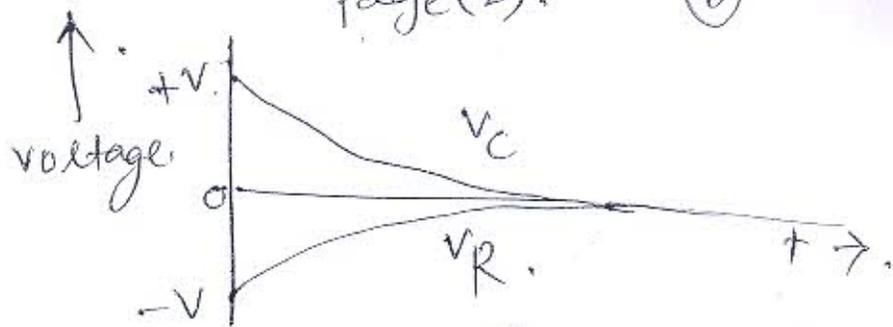
at  $t=0$ ,  $i = -\frac{V}{R}$ , equ<sup>n</sup> (7) becomes.

$$-\frac{V}{R} = Ke^{\frac{0}{RC}}$$

$\Rightarrow$   $K = -\frac{V}{R}$  putting this value in equ<sup>n</sup> (7)

$$i = -\frac{V}{R} e^{-t/RC} = -I_0 e^{-t/RC}$$

$$i = -I_0 e^{-t/RC} \quad \text{--- (8)}$$



$$V_R = Ri = -I_0 R e^{-t/\tau} = -V e^{-t/\tau}$$

$$V_C = \frac{1}{C} \int i \cdot dt = \frac{1}{C} \int -I_0 e^{-t/\tau}$$

$$\Rightarrow \boxed{V_C = I_0 R e^{-t/\tau} = V e^{-t/\tau}}$$

The charge in capacitor during discharging is.

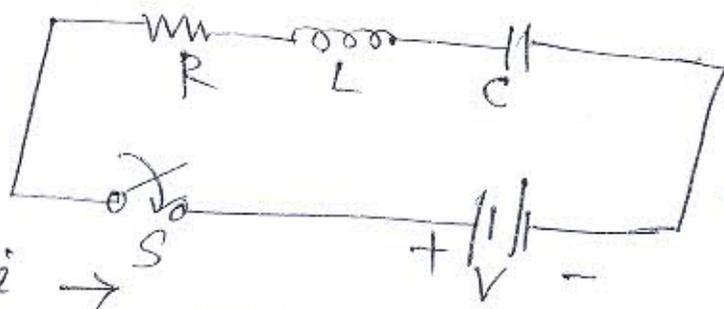
$$q = CV_C = C V e^{-t/\tau} = Q e^{-t/\tau}$$

where  $Q = CV = \text{maximum charge in}$

Capacitor.

Transient Response of second order

Solu<sup>n</sup> of second order circuit.



Two types of energy  
electromagnetic  
electrostatic

Applying KVL.

$$iR + L \frac{di}{dt} + \frac{1}{C} \int i \cdot dt = V$$

$$\Rightarrow R \cdot \frac{di}{dt} + L \frac{d^2 i}{dt^2} + \frac{i}{C} = 0$$

$$\Rightarrow \frac{d^2 i}{dt^2} + \frac{R}{L} \cdot \frac{di}{dt} + \frac{1}{LC} i = 0 \quad \text{--- (1)}$$

Equ<sup>n</sup> (1) is a second order linear differential equ<sup>n</sup>.

Its characteristics equ<sup>n</sup> is.

$$\boxed{P^2 + \frac{R}{L}P + \frac{1}{LC} = 0} \quad \text{--- ②} \quad \left[ \because P = \frac{d}{dt} \right]$$

The roots of this char. equ<sup>n</sup>

$$= -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

$$= \alpha \pm \beta.$$

$$P_1 = \alpha + \beta, \quad P_2 = \alpha - \beta.$$

where  $\alpha = -\frac{R}{2L}$  and  $\beta = \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$ .

The sol<sup>n</sup> of the differential equ<sup>n</sup> is,

$$\boxed{i = K_1 e^{P_1 t} + K_2 e^{P_2 t}} \quad \text{--- ③}$$

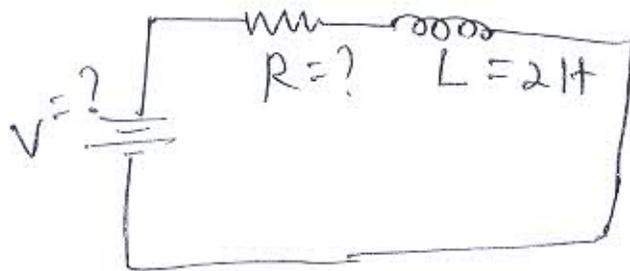
where  $K_1$  and  $K_2$  are constants whose values are calculated from boundary conditions.

Solved Examples :: →

Ex-1 A constant voltage is applied to R-L series circuit at  $t=0$ , by closing a switch. The voltage across 'L' is 25V at  $t=0$  and drops to 5V at  $t=0.025$  sec.  $L=2H$ . find.

- (a) the applied voltage.
- (b) the value of R

Soln



(a) At  $t=0$ , the entire voltage is dropped across inductor and no voltage is dropped across the resistor. so applied voltage  $V=25$  volt.

(b) At any instant voltage across inductor is 5V  
 $V_L = V e^{-t/\tau}$  ( $V_L = 5$  volt)

$$\Rightarrow 5 = 25 e^{-\frac{0.025}{\tau}}$$

$$\Rightarrow \tau = 0.01553$$

$$\Rightarrow \frac{L}{R} = 0.01553$$

$$\Rightarrow R = \frac{L}{0.01553} = \frac{2}{0.01553} = 128.78 \Omega$$

$$\Rightarrow \boxed{R = 128.78 \Omega}$$

Exa 2 The winding of an electromagnet has an inductance of  $3\text{H}$  and resistance of  $15\Omega$  when it is connected to  $120\text{V}$  dc supply. Calculate.

- (a) The steady state value of current flowing in the winding.
- (b) The time constant of the circuit.
- (c) The value of induced emf after  $0.1\text{ sec}$ .
- (d) The time for the current to rise to  $85\%$  of its final value.
- (e) The value of current after  $0.3\text{ sec}$ .
- (a) Given.  $R = 15\Omega$ ,  $L = 3\text{H}$  and  $V = 120\text{V}$   
 steady state current  $I_0 = \frac{V}{R} = \frac{120}{15} = 8\text{A}$
- (b)  $\tau = \frac{L}{R} = \frac{3}{15} = 0.2\text{ sec}$ .
- (c) The value of induced emf after  $0.1\text{ sec}$  is  
 $V_L = V e^{-t/\tau} = 120 \left( e^{-\frac{0.1}{0.2}} \right) = 72.783\text{ volt}$ .
- (d) Current at any instant during rise is,  
 $i = I_0 \left( 1 - e^{-\frac{t}{\tau}} \right)$   
 $\Rightarrow \frac{85}{100} I_0 = I_0 \left( 1 - e^{-\frac{t}{\tau}} \right)$   
 $\Rightarrow \boxed{t = 0.379\text{ sec}}$
- (e)  $i = I_0 \left( 1 - e^{-\frac{t}{\tau}} \right) = 8 \left( 1 - e^{-\frac{0.3}{0.2}} \right)$   
 $\Rightarrow \boxed{i = 6.21\text{ A}}$

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Exa-3 An.  $8\mu\text{F}$  capacitor is connected in series with  $0.5\text{ M}\Omega$  resistance across  $200\text{V}$  supply. Calculate (a) initial charging current (b) the current and p.d. across capacitor, 4 seconds after it is connected to the supply.

Solution given  $C = 8\mu\text{F} = 8 \times 10^{-6}\text{ F}$ .

$$R = 0.5\text{ M}\Omega = 0.5 \times 10^6 \Omega$$

$$V = 200\text{ Volt}$$

(a) initial charging current  $= \frac{V}{R}$   
 $= \frac{200}{0.5 \times 10^6} = 4 \times 10^{-4}\text{ A}$

(b) Time constant  $= \tau = RC = 0.5 \times 10^6 \times 8 \times 10^{-6}$   
 $= 4\text{ sec}$ .

\* The current across capacitor is  $4\text{ sec}$ , after it is connected to the supply is.

$$i = \frac{V}{R} e^{-t/\tau} \Rightarrow i = 4 \times 10^{-4} e^{-\frac{4}{4}}$$

$$\Rightarrow i = 1.47 \times 10^{-4}\text{ A}$$

\* The p.d. across capacitor in  $4\text{ seconds}$  after it is connected to the supply is.

$$V_c = V (1 - e^{-t/\tau})$$

$$\Rightarrow V_c = 200 (1 - e^{-4/4}) = 126.44\text{ Volt}$$

Exa-4 The time constant of a coil was found to be 2.5 ms. With a resistance of  $100\Omega$  added in series, a new time constant of 0.5 ms was obtained. Find R and L of the coil.

Solution Given time constant  $\tau = \frac{L}{R} = 2.5 \text{ ms}$

→ When resistance (R) added, then time constant =  $2.5 \times 10^{-3} \text{ s}$

$$\tau' = \frac{L}{R+100} = 0.5$$

$$\frac{\tau}{\tau'} = \frac{2.5}{0.5}$$

$$\Rightarrow \frac{\frac{L}{R}}{\frac{L}{R+100}} = 5$$

$$\Rightarrow \frac{R+100}{R} = 5$$

$$\Rightarrow \boxed{R = 25\Omega}$$

But given  $\frac{L}{R} = 2.5 \times 10^{-3}$

$$\text{So } L = 2.5 \times 10^{-3} \times R = 2.5 \times 10^{-3} \times 25$$

$$= 62.5 \times 10^{-3} \text{ H}$$

$$\Rightarrow \boxed{L = 62.5 \times 10^{-3} \text{ H}}$$

# Initial and steady state conditions

Sr. NO	Name of the element.	Symbol	At, $t=0$	At, $t=\infty$
01.	Resistance.			
02.	Inductance.			
03.	Capacitance.			
04.	Inductance with initial current $I_0$ .			
05.	Capacitance with initial voltage $V_0$ .			

## Time constants.

$R-L$  ckt  $\rightarrow Z = \frac{L}{R} = \text{Time constant.}$   
 or inductive time constant.

$R-C$  ckt  $\rightarrow Z = R \cdot C = \text{Time constant}$   
 or capacitive time constant.

Exa-5 A coil having  $R=1 \text{ ohm}$  and  $L=500 \text{ mH}$  is switched on to a voltage source of  $3 \text{ V d.c.}$

- Determine:
- Initial and final value of current
  - time constant of the circuit.
  - Current at  $t = 500 \text{ ms}$ .